Section 7

Instrumental Variables

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GOV 2003

Oct 21, 2021

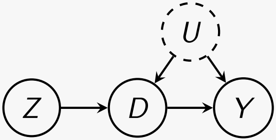
Overview

- Logistics:
  - **Pset 6 released!** Due at 11:59 pm (ET) on Oct 27

- Today's topics:
  1. Noncompliance in randomized experiments
  2. IV in observational studies using TSLS

Noncompliance in randomized experiments

- Motivation: What if there is unmeasured confounding?
- In randomized experiments: when treatment assignment is randomized but cannot intervene treatment uptake.
  - $\rightsquigarrow$ **noncompliance** (one- or two-sided)
- DAG example:



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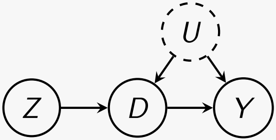

- Estimand: LATE = ITT effect on the outcome for compliers

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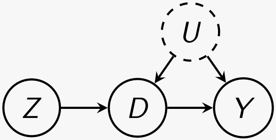

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  - Q: Why not ITT_Y?
- Example (one- or two-sided)? Identification? Estimation?

Noncompliance (one-sided)

- One-sided example:
  canvass assignment (Z_i) - canvass recieved (D_i) - turn out (Y_i)
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- Compliance type by $D_i(1)$:
  - $D_i(1) = 1$: Compliers. If assigned to canvassing, I **would** recieve it.
  - $D_i(1) = 0$: Noncompliers. If assigned to canvassing, I **would not** receive it.
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- Assumptions:
  1. Randomization of Z_i
  2. Presence of some compliers $\pi_{\text{co}} \neq 0$
  3. Exclusion restriction $Y_i(z, d) = Y_i(z', d)$ (i.e., Z_i only affects Y_i through D_i)

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  - **Wald** or **IV estimator**

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  - Equivalent to the **TSLS estimator** under binary instrument and binary treatment
    1. Regress D_i on Z_i and get fitted values $\widehat{D}_i$
    2. Regress Y_i on $\widehat{D}_i$ and get the slope
    - Intuitively, TSLS retains only the variation in D_i that is generated by the instrument Z_i in the first stage.
    - $\rightsquigarrow$ use `AER::ivreg()` in practice.

Noncompliance (one-sided)

- Variance estimation:
  - **Wald** estimator: Use delta method to find the asymptotic variance

$$\mathbb{V}[\widehat{\tau}_{iv}] \approx \frac{1}{\text{ITT}_D^2}\mathbb{V}\left[\widehat{\text{ITT}}_Y\right] + \frac{\text{ITT}_Y^2}{\text{ITT}_D^4}\mathbb{V}\left[\widehat{\text{ITT}}_D\right] - 2\frac{\text{ITT}_Y}{\text{ITT}_D^3}\text{cov}\left[\widehat{\text{ITT}}_Y, \widehat{\text{ITT}}_D\right]$$

  - **TSLS** estimator: **Don't** use SEs from second step (see MHE section 4.6.1 2SLS Mistakes) $\rightsquigarrow$ use `ivpack::robust.se()` in practice.

Noncompliance (two-sided)

- Two-sided example:
  encouragement (Z_i) - treatment (D_i) - outcome (Y_i)
  - Or, testing habitual voting (Coppock and Green 2016): GOTV canvassing (2006) - turn out (2006) - turn out (2008)
  - LATE: Habitual voting for those who would vote iif they are contacted by a canvasser in this election

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- Compliance type by $(D_i(0), D_i(1))$:
  - (0,1): Complier
  - (1,1): Always-taker
  - (0,0): Never-taker
  - (1,0): Defier
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  - Q: Can we identify this by observing Z_i and D_i? What can we know (hint: ITT_D)?
- Assumptions: 1-3 from the previous setup, and
  4. Monotonicity: $D_i(1) \geq D_i(0), \forall i$ (no defiers)
  - Q: What does exclusion restriction/monotonicity imply in words?

Noncompliance (two-sided)

- Same identification result:

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- Further issues:
  - What if exclusion restriction/monotonicity is violated? Can we still use IV estimand for LATE? [Pset 6 Q2]
  - Detecting weak instruments? [Pset 6 Q3 (c)]

In R: Wald estimator

```r
# *Recall what we did in Neyman's approach*
my_data # data includes Z, D, and Y

# Proportion of compliers (using ITT_D)
pi_co <- mean(my_data$D[my_data$Z == 1]) - mean(my_data$D[my_data$Z == 0])

# Compute ITT's
ITT_Y <- mean(my_data$Y[my_data$Z == 1]) - mean(my_data$Y[my_data$Z == 0])
ITT_D <- mean(my_data$D[my_data$Z == 1]) - mean(my_data$D[my_data$Z == 0])
# (ITT_D = pi_co)

# TODO 1: Compute Wald estimator
Wald_est <- NULL
```

In R: Wald estimator

```
# TODO 2: Compute variance
# TODO 2-1: Compute variance terms using neyman estimator
Var_ITT_Y_est <- NULL
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# Compute covariance term
# demean
demeaned_y <- my_data$Y[my_data$Z == 1] - mean(my_data$Y[my_data$Z == 1])
demeaned_d <- my_data$D[my_data$Z == 1] - mean(my_data$D[my_data$Z == 1])
# denominator
denom <- sum(my_data$Z)*(sum(my_data$Z) - 1)
Covar_est <- (demeaned_y %*% demeaned_t)/denom

# TODO 2-2: Compute the estimate of the formula in p.6
Var_Wald_est <- NULL
```

In R: TSLS estimator

```
ivmodel <- AER::ivreg(Y ~ D | Z, data = my_data)
ivpack::robust.se(ivmodel)
```

IV in observational studies using TSLS

- Motivation: What if there is unmeasured confounding?
- In observational studies: In case where
  - treatment is not randomized and there exist unmeasured confounder;
  - can find instrumental variable;
  - exogenous covariates ($\mathbf{X}_i$): may exist observable confounders between Z_i, D_i, and Y_i
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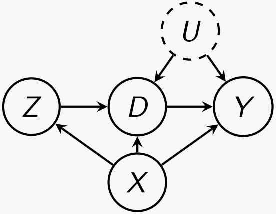

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  - w/o covariates

$$D_i = \delta + \gamma Z_i + \eta_i$$
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- Recall the four canonical IV assumptions.
- Suppose we have binary treatment and binary instrument.
  - w/o covariates, TSLS estimand (τ) = LATE (τ_{LATE}) and TSLS estimator = Wald estimator

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- Recall the four canonical IV assumptions.
- Suppose we have binary treatment and binary instrument.
  - w/o covariates, TSLS estimand (τ) = LATE (τ_{LATE}) and TSLS estimator = Wald estimator
  - w/ covariates, we need **constant effects** so that TSLS estimand (τ) = LATE (τ_{LATE})
    - Otherwise, τ is an odd weighted function of causal effects

In R: TSLS estimator w/ exogenous covariates

```
ivmodel <- AER::ivreg(Y ~ X1 + X2 + D | X1 + X2 + Z, data = my_data)
# Or, equivalently: Y ~ X1 + X2 + D | . -D + Z
ivpack::robust.se(ivmodel)
# For clustered error: use ivpack::cluster.robust.se()
```