Module 8: Regression Discontinuity Designs

Fall 2021

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Gov 2003 (Harvard)

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- Regression discontinuity: a discontinuity in treatment assignment

Plan of attack

1. Sharp Regression Discontinuity Designs

2. Estimation in the SRD

3. Bandwidth selection

4. Fuzzy Regression Discontinuity Designs

1/ Sharp Regression Discontinuity Designs

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  - Class size on test scores using total student thresholds to create new classes (Angrist and Lavy, 1999)

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  - Can't use standard identification toolkit for ATE/ATT.

Plotting the RDD (Imbens and Lemieux, 2008)

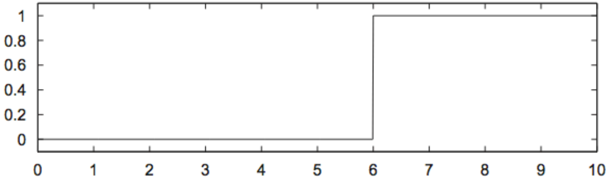

Fig. 1. Assignment probabilities (SRD).

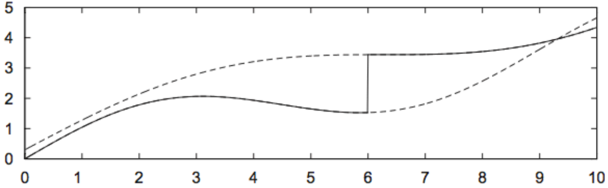

Fig. 2. Potential and observed outcome regression functions.

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  - Extrapolation requires **smoothness**

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- Note that this is the same for the treated group:

$$E[Y_i(1)|X_i = c] = \lim_{x \downarrow c} E[Y_i|X_i = x]$$

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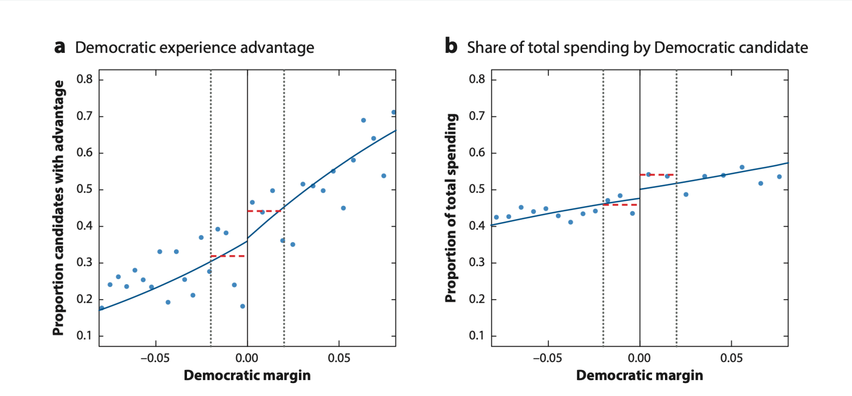
Problems with local randomization assumptions



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  - Students with more money $\rightsquigarrow$ more exam retaking $\rightsquigarrow$ sorting.

2/ Estimation in the SRD

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  - b_k are the bin cutpoints.
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- What to observe:
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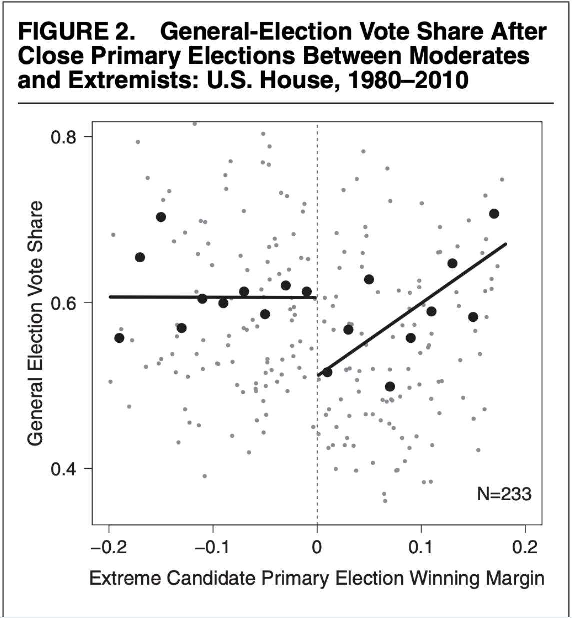
  - If it's not in the binned mean plot, unlikely to be a robust/credible effect.

Example from close election RD



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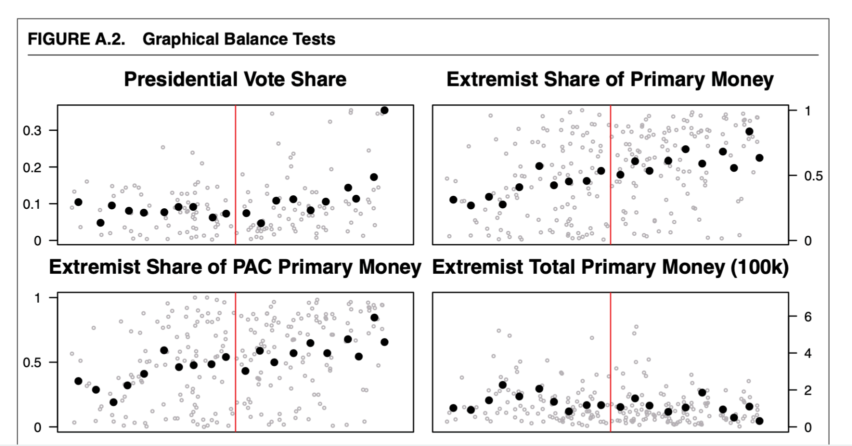
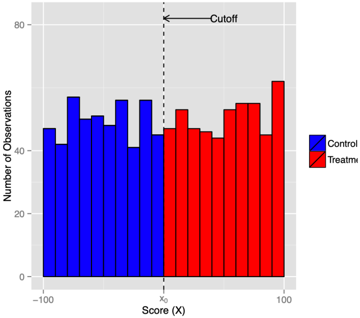
Checking covariates at the discontinuity

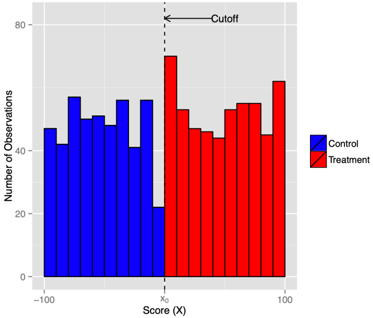


McCrary Test

Figure 10: Histogram of Score



(a) No sorting



(b) Sorting

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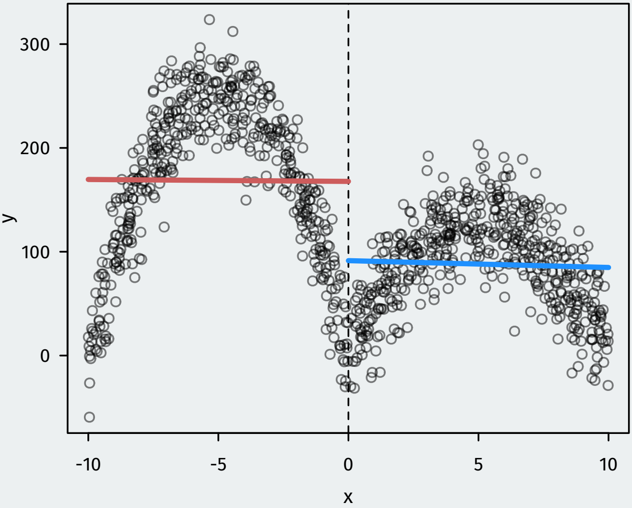
Example of misleading trends



Nonparametric and semiparametric approaches

- Upper and lower limit functions:

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- Basically means among units no more than h away from the threshold.

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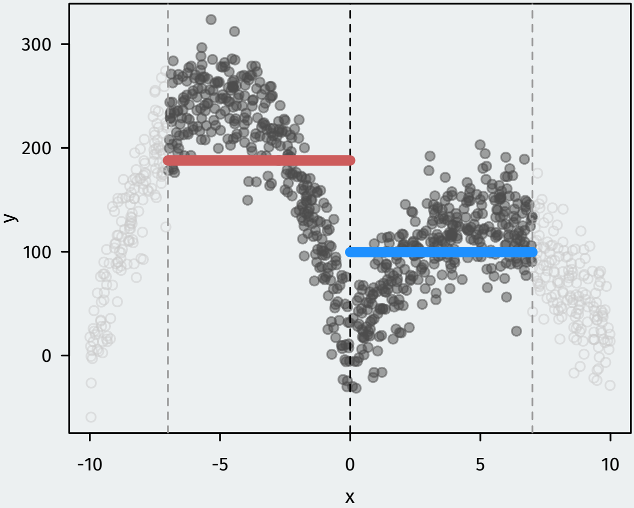
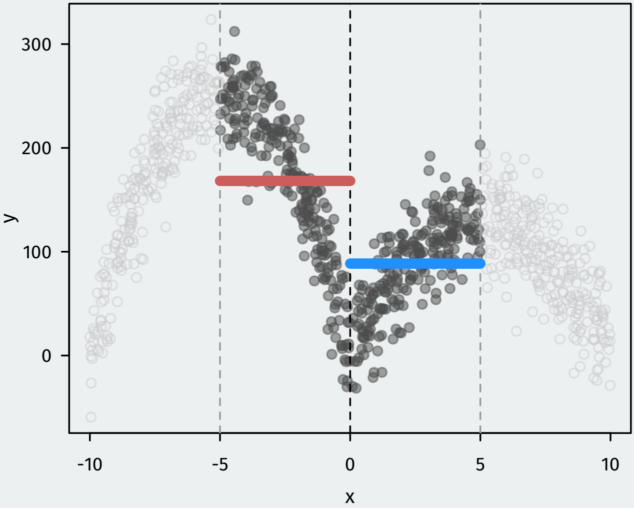
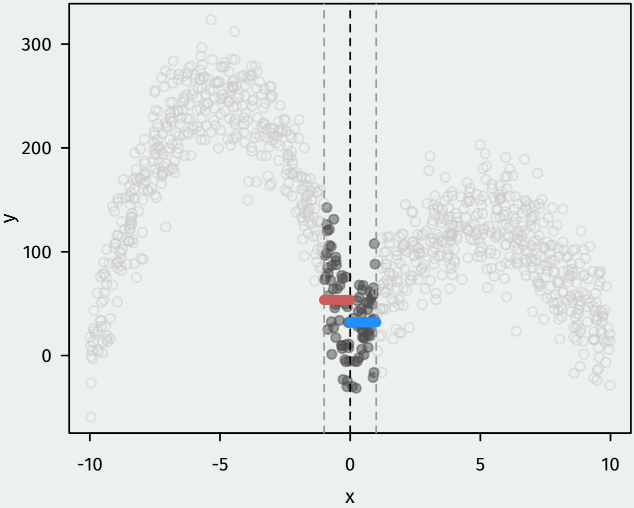


Bandwidth equal to 5



Bandwidth equal to 1



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    - Likely large finite sample bias, poor coverage of confidence intervals.

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- Our estimate is

$$\begin{aligned}
\widehat{\tau}_{\text{SRD}} &= \hat{\mu}_+(c) - \hat{\mu}_-(c) \\
&= \hat{\alpha}_+ + \hat{\beta}_+(c-c) - \hat{\alpha}_- - \hat{\beta}_-(c-c) \\
&= \hat{\alpha}_+ - \hat{\alpha}_-
\end{aligned}$$

More practical estimation

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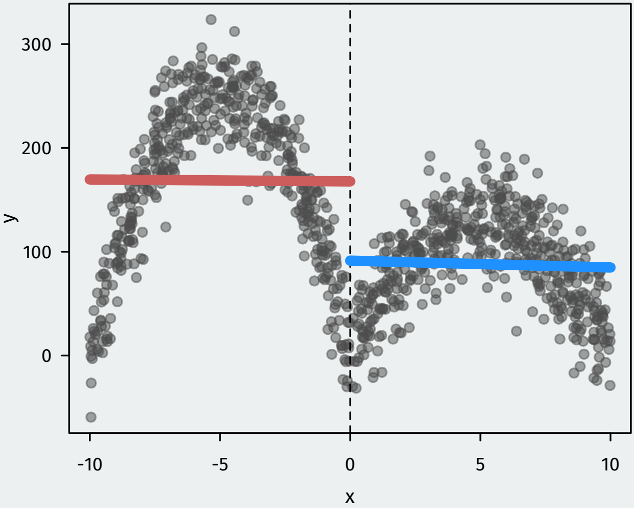
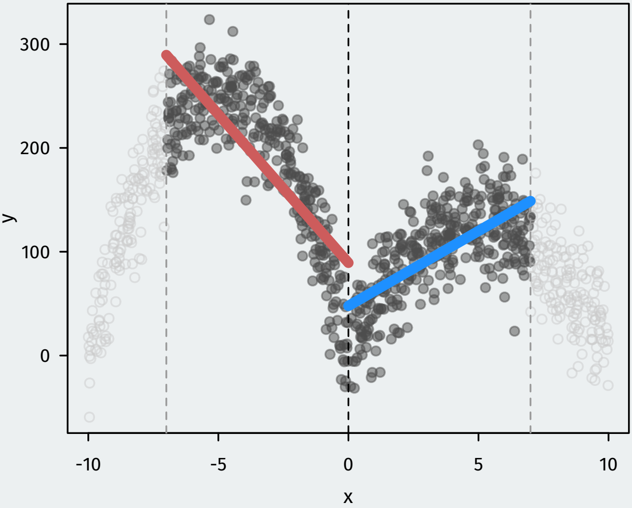
  - Popular choice is the **triangular kernel**: $K(u) = (1 - |u|) \cdot \mathbf{1}(|u| < 1)$

Bandwidth equal to 10 (Global)



Bandwidth equal to 7



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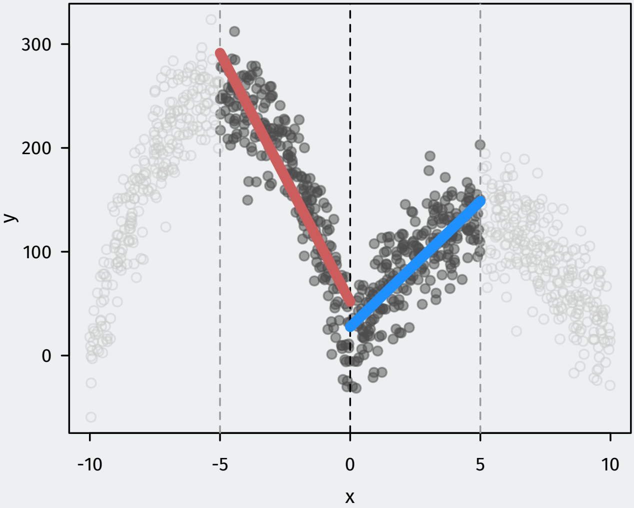

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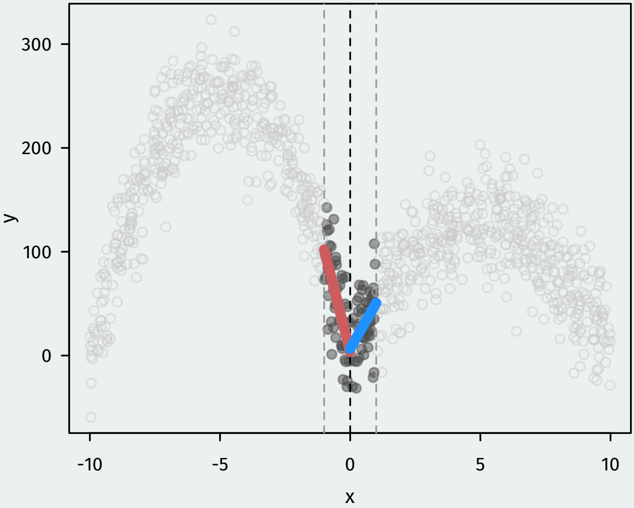

3/ Bandwidth selection

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Bandwidths and bias

- Optimal bandwidth shrinks fast enough so $h_n \propto n^{-1/5}$.
    - But this results in asymptotic bias, two possible solutions.
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- Coverage of CIs can be very bad without RBC!

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- Possible to use local polynomial regression beyond linear, but performance is poor (very sensitive to end points)
- Use `{rdrobust}` package for CCT bandwidths/estimation.

4/ Fuzzy Regression Discontinuity Designs

Setup

- **Fuzzy RD**: discontinuity in the probability of treatment.

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  - affects Y_i, but only through D_i (at the threshold)

Fuzzy RD in a graph

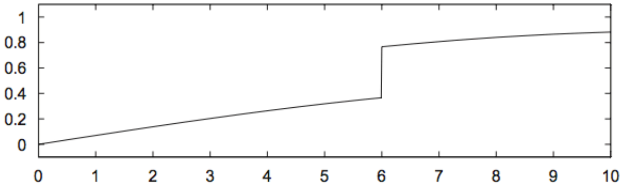

Fig. 3. Assignment probabilities (FRD).

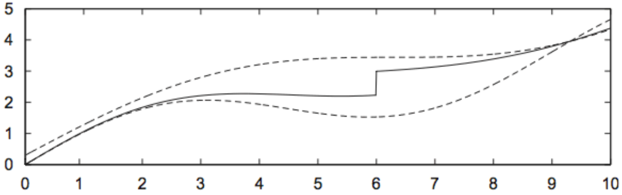

Fig. 4. Potential and observed outcome regression (FRD).

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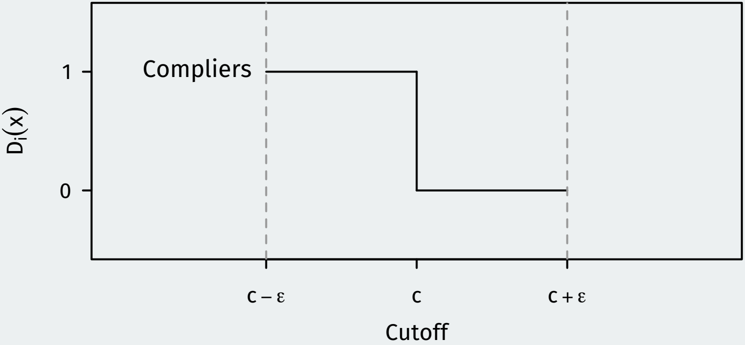

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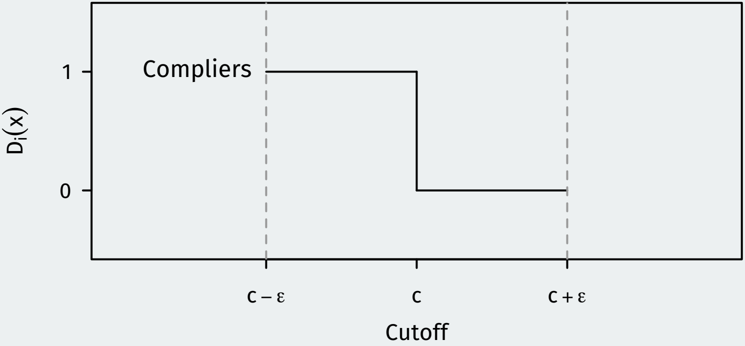

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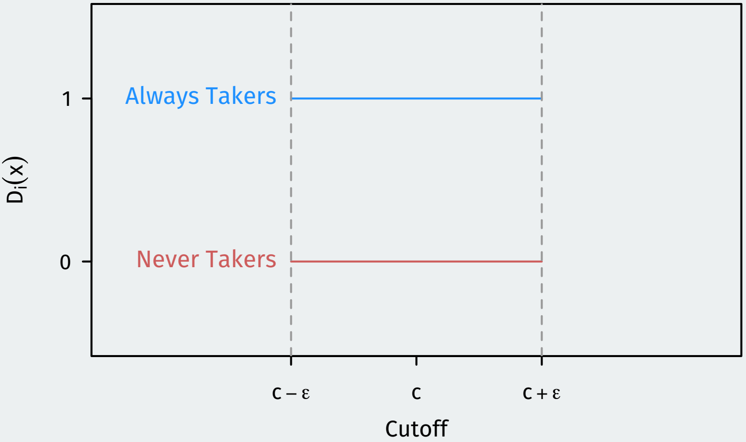

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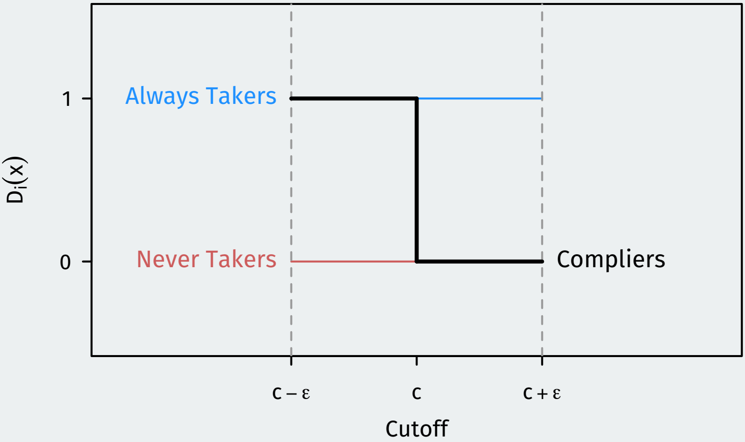

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- We can define an estimator that is in the spirit of IV:

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- Thus, being above the threshold is treated like an instrument, controlling for trends in X_i.

Kink RD

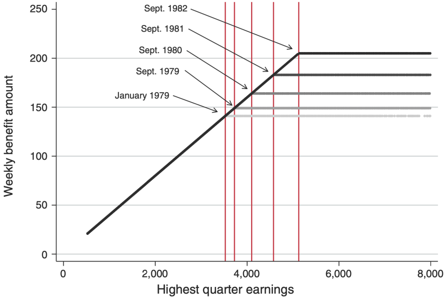

FIGURE 2. LOUISIANA: SCHEDULE OF UI WEEKLY BENEFIT AMOUNT, JAN. 1979–DEC. 1983

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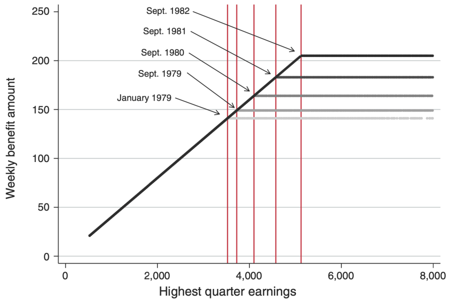
Kink RD



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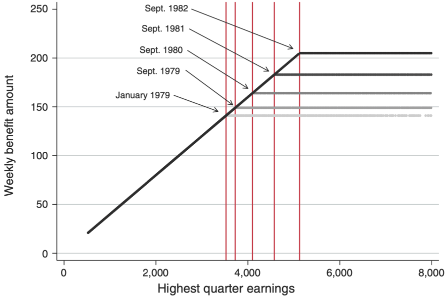

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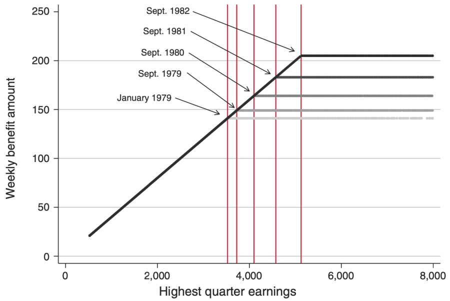

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    - Estimation Similar, but better to use local quadratic regression.