Module 5: Observational Studies

Fall 2021

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Gov 2003 (Harvard)

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- Up to now: experiments where design makes everything easier.
- Now: what happens when do observational studies?
    - Start with identification, selection on observables, and DAGs.
    - Rest of the course will cover different designs for observational studies.

1/ Identification in observational studies

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   - Sometimes written as $D_i \perp\!\!\!\perp (\mathbf{Y}(1), \mathbf{Y}(0))$

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- **Selection bias**: how different the treated and control groups are in terms of their potential outcome under control.

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- With an unbounded Y_i, we can't even bound the ATT because, in principle, SB could be anywhere from $-\infty$ to ∞.
- We say ATT (and ATE) are **unidentified** without further assumptions.

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  - Or you will have justify them through argument.

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  - Keep them separate: estimator shouldn't drive identification.

What is confounding?

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2/ Selection on observables

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- Key part of the above proof:

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- These make two very different assumptions about the CEFs!

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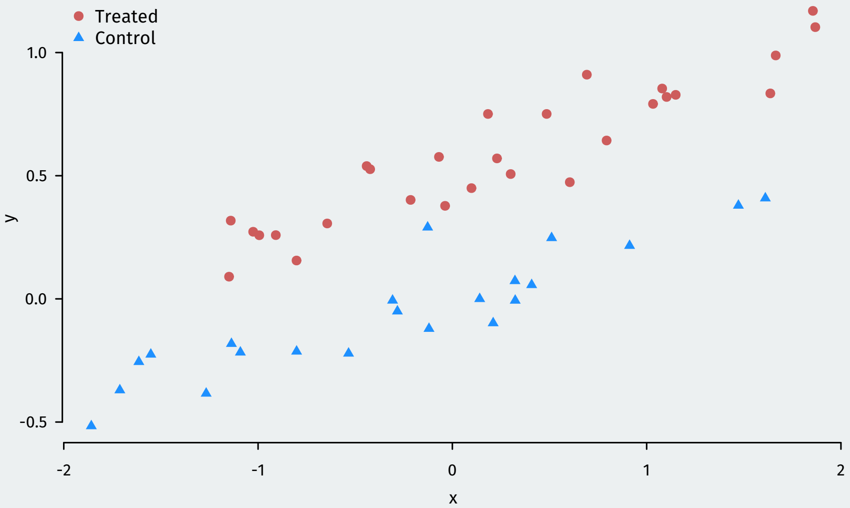
Imputation estimator visualization



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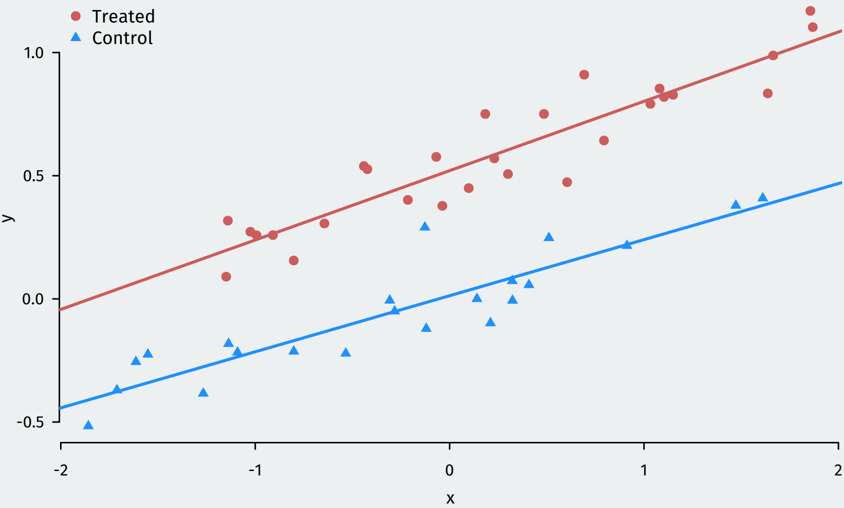

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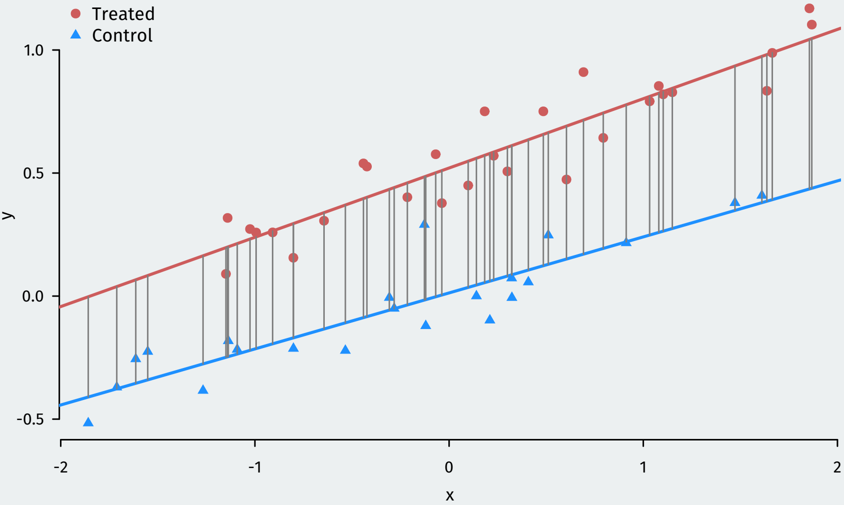

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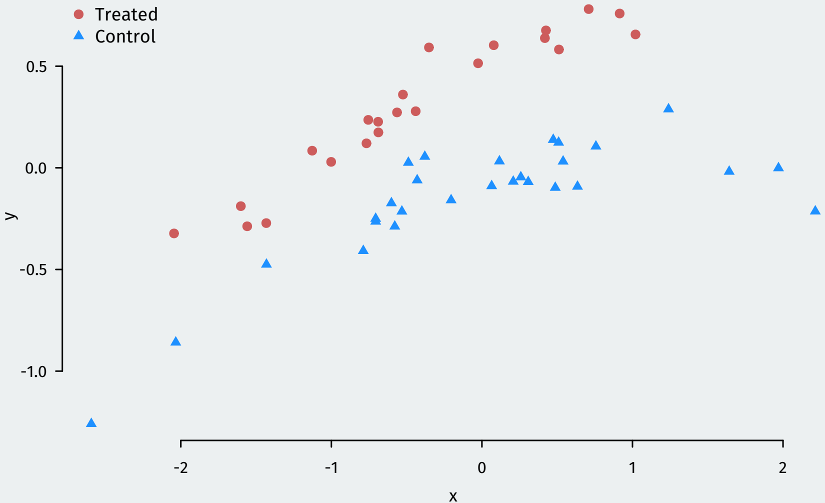

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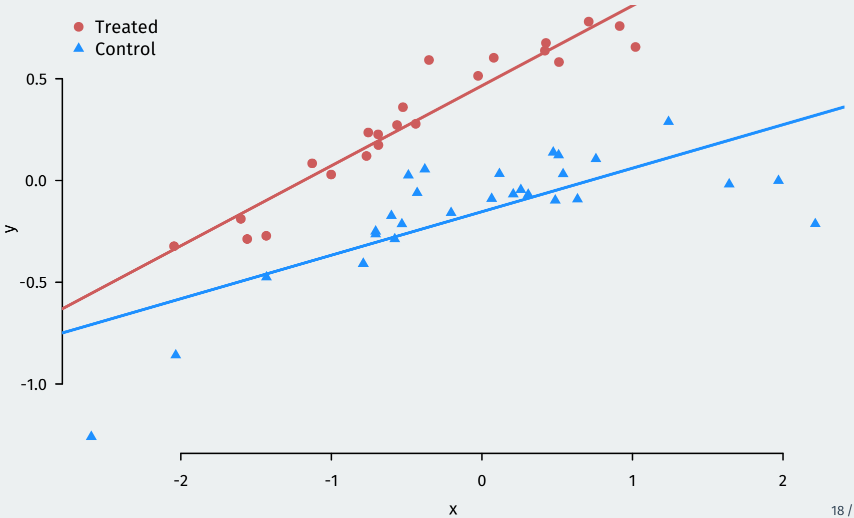

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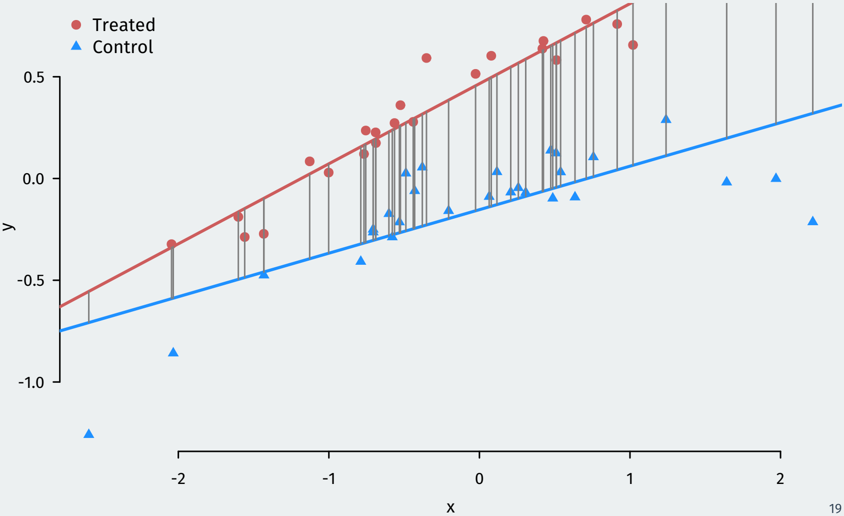

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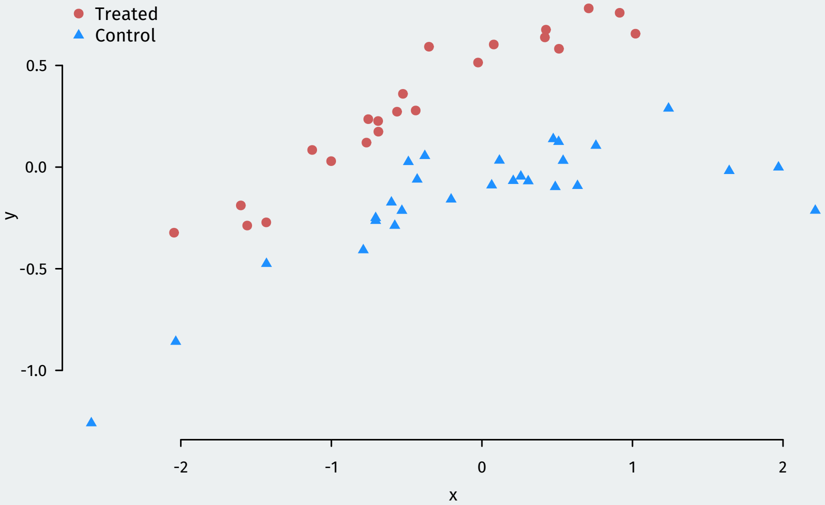
```
library(mgcv)
mod0 <- gam(y~s(x), subset = d==0)
summary(mod0)
```

```
##
## Family: gaussian
## Link function: identity
##
## Formula:
## y ~ s(x)
##
## Parametric coefficients:
##             Estimate Std. Error t value Pr(>|t|)
## (Intercept)   -0.154      0.019    -8.1  5.1e-08 ***
## ---
## Signif. codes:
## 0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Approximate significance of smooth terms:
##       edf Ref.df    F p-value
## s(x) 5.17   6.29 36.9  <2e-16 ***
## ---
```

Using GAMs



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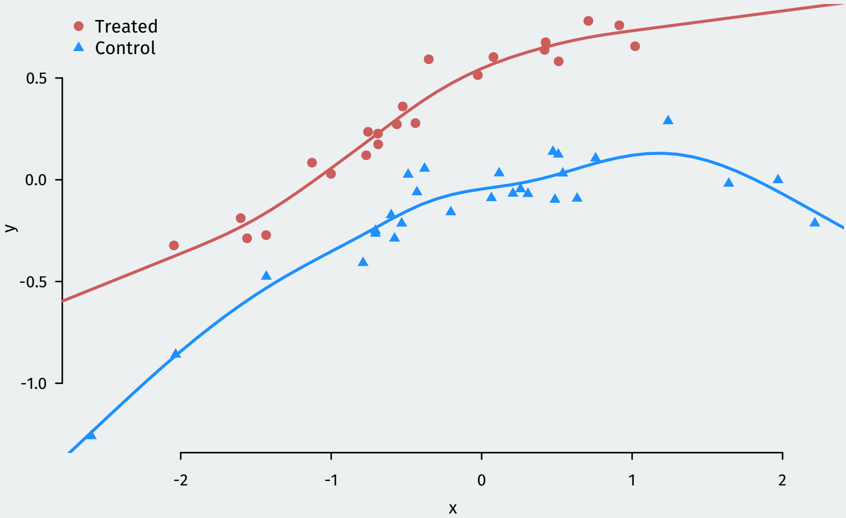

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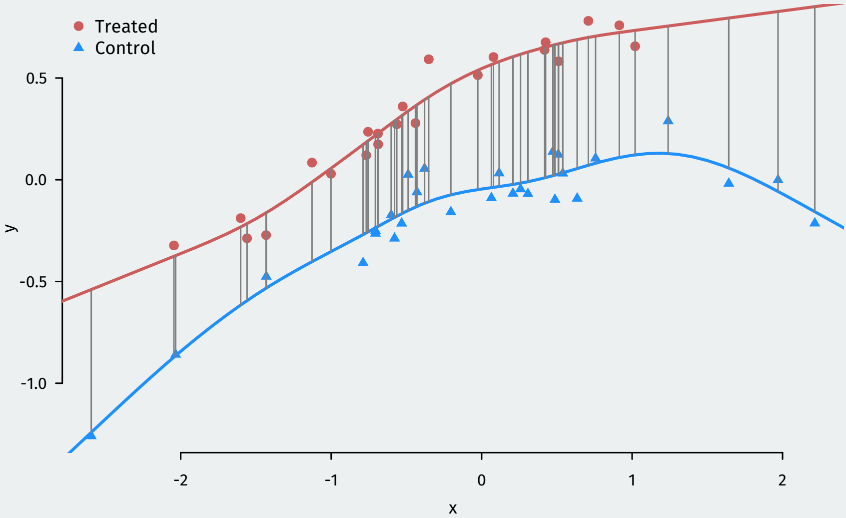

3/ DAGs

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- Another way: use DAGs and look at back-door paths.

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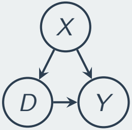

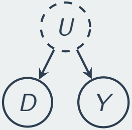

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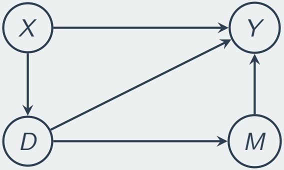

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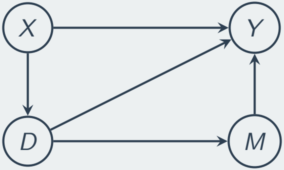

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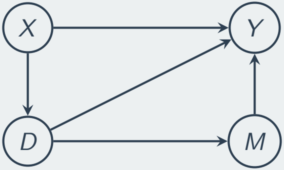

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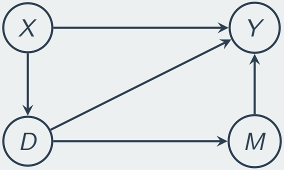

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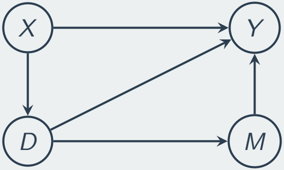

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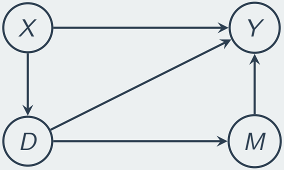

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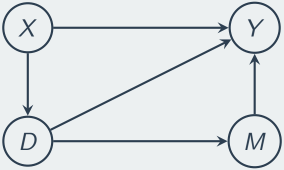

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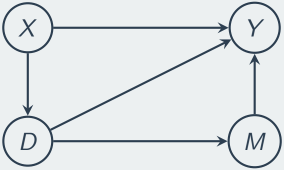

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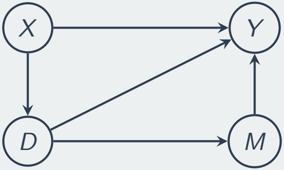

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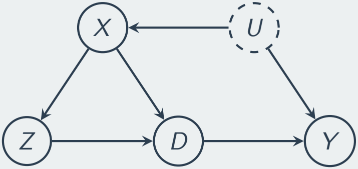
DAGs to distributions



$$Y = f_y(D, U, \varepsilon_y)$$
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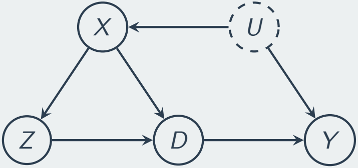

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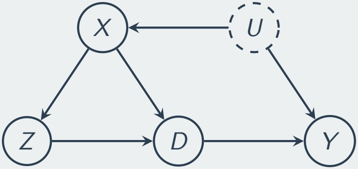

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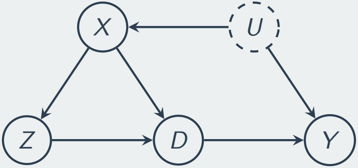

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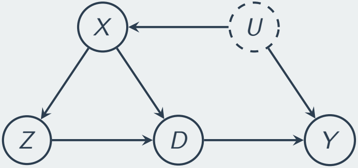

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- Causal DAGs imply the following factorization (some conditions apply):

$$\mathbb{P}(X_1, X_2, \dots, X_J) = \prod_{j=1}^{J} \mathbb{P}(X_j \mid \text{pa}(X_j)) \quad \text{where} \quad \text{pa}(X_j) = \text{parents of } X_j$$

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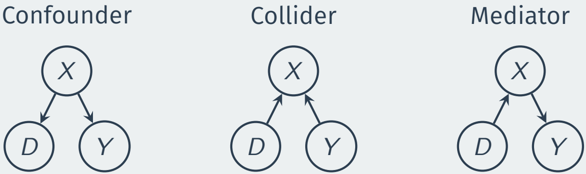

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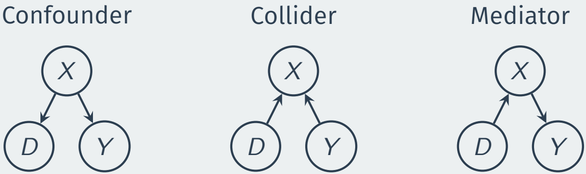

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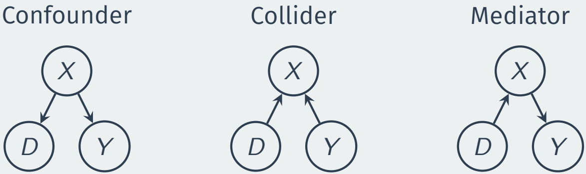

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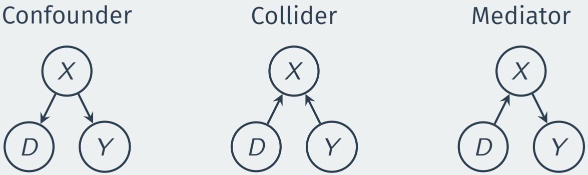

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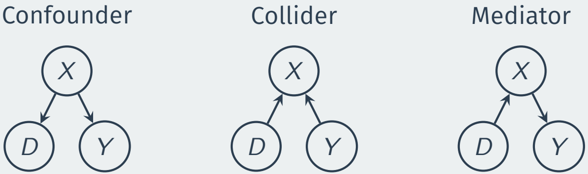

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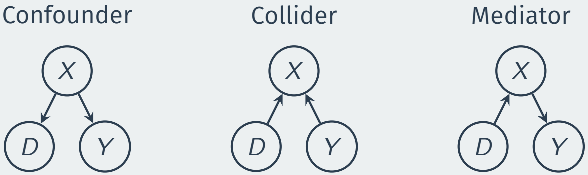

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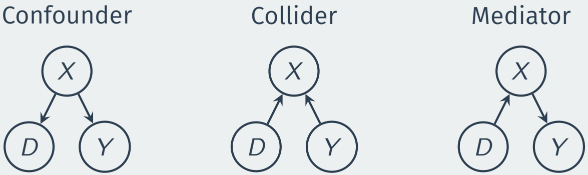

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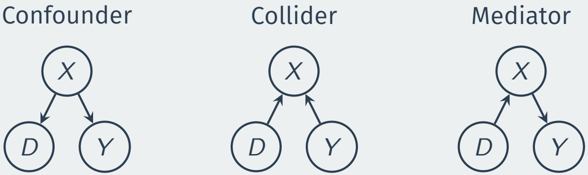

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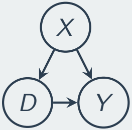

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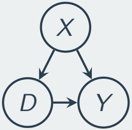
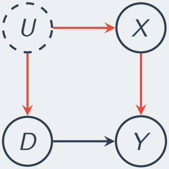


- Here: backdoor path $D \leftarrow X \rightarrow Y$

Other types of confounding



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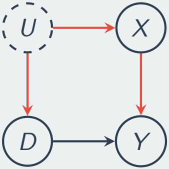

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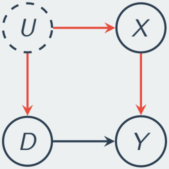

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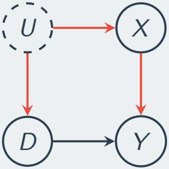

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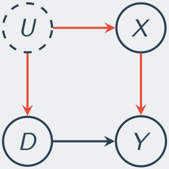

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- The backdoor criterion is fairly powerful. Tells us:
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  - what variables to condition on to eliminate the confounding.

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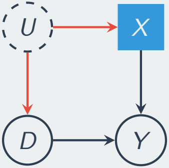

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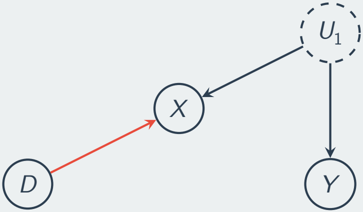

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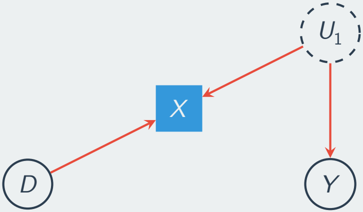

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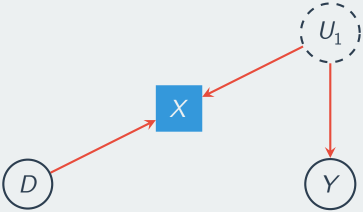

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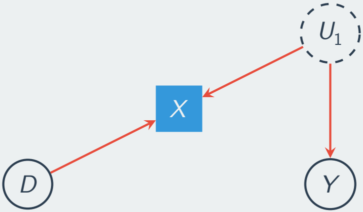

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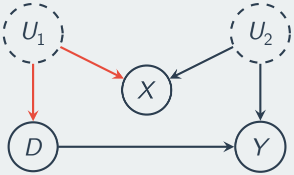
  - But still no causal relationship $\rightsquigarrow$ selection bias.

M-bias



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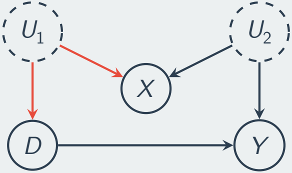

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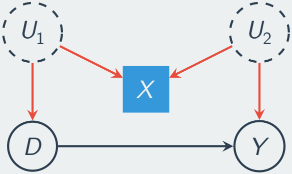

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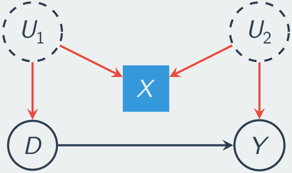

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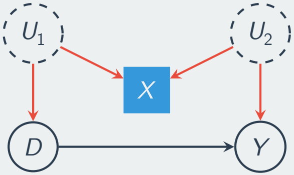

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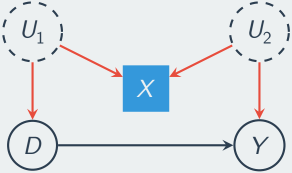

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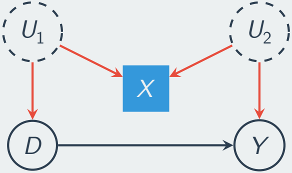

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    - Pearl and others think M-bias is a real threat.

4/ Sensitivity analysis

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    2. **Partial identification**: abandon point identification and try to find bounds for the ATE under different assumptions.

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$$Y_i = \alpha + \tau D_i + \mathbf{X}_i' \beta + \gamma U_i + \varepsilon_i \qquad \widehat{\tau} \overset{p}{\to} \tau + \gamma \times \underbrace{\frac{\text{cov}(D_i^{\perp \mathbf{X}}, U_i^{\perp \mathbf{X}})}{\mathbb{V}(D_i^{\perp \mathbf{X}})}}_{\text{regression of } U_i^{\perp \mathbf{X}} \text{ on } D_i^{\perp \mathbf{X}}}$$

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- Standard **omitted variable bias formula**:

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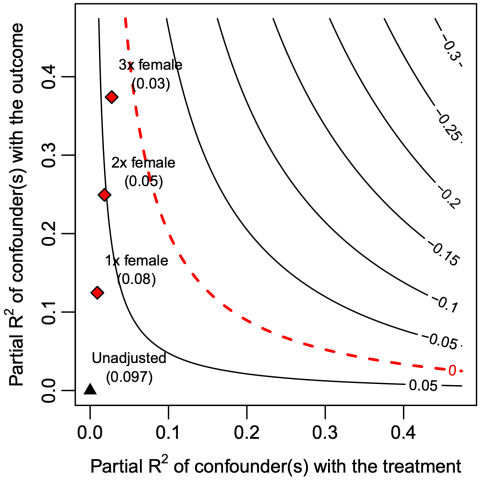
    - From these we can determine the bias and thus the true value of τ

Sensitivity analysis example



5/ Partial identification and bounds

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- Can we improve using data? Rewrite the ATE with $p = \mathbb{P}(D_i = 1)$:

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  - But always will contain 0. Weak assumptions $\rightsquigarrow$ weak inferences

Optimized treatment choice

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